Matroids
Matroids

Start with a set of objects, for example: $E = \{1, 2, 3, 4, 5\}$
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The power set of $E$ is the set of all possible subsets of $E$:

$$
\{\}, \\
\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \\
\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \\
\{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \\
\{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 3, 4\}, \{1, 3, 5\}, \\
\{1, 4, 5\}, \{2, 3, 4\}, \{2, 3, 5\}, \{2, 4, 5\}, \{3, 4, 5\}, \\
\{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{2, 3, 4, 5\}, \\
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A matroid is a subset \( I \) of the power set of \( E \) such that

1. For all \( I_1 \in I \), if \( I_2 \subset I_1 \) then \( I_2 \in I \).
2. For all \( I_1, I_2 \in I \) s.t. \( |I_2| = |I_1|+1 \), \( \exists e \in I_2 \) s.t. \( I_1 \cup \{e\} \in I \).
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Matroids

Start with a set of objects, for example: \( E = \{1, 2, 3, 4, 5\} \)
Now put weights on elements, subset weight = sum of weights of its elements.

\[
\emptyset, \\
\{1\}, \{2\}[5], \{3\}[6], \{4\}[12], \{5\}[8], \\
\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}[11], \\
\{2,4\}[17], \{2,5\}[13], \{3,4\}[18], \{3,5\}, \{4,5\}[20], \\
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Matroids

Start with a set of objects, for example: \( E = \{1, 2, 3, 4, 5\} \)
Consider: Find max card subset of min (max) weight

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Matroids

How to solve it?

Given: Matroid \((E,I)\) with weights on elements of \(E\)
Find: Maximum Cardinality Independent Set of \(I\) with minimum (maximum) weight

Set \(T = \emptyset\);
Order all elements in \(E\) by increasing (decreasing) weight;
Repeat the following until \(E\) is empty:
   Let \(e\) be the first element of \(E\);
   Remove \(e\) from \(E\) (pop it off);
   If \(T \cup \{e\} \in I\) then Set \(T = T \cup \{e\}\);
Output \(T\);
Matroids

{},
{1}[], {2}[5], {3}[6], {4}[12], {5}[8],
{1,2}[], {1,3}[], {1,4}[], {1,5}[], {2,3}[11],
{2,4}[17], {2,5}[13], {3,4}[18], {3,5}[], {4,5}[20],
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{1,2,3,4,5}[]

Order:  {2}[5], {3}[6], {5}[8], {4}[12]
Matroids

{},
{1}[], {2}[5], {3}[6], {4}[12], {5}[8],
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{1,2,3,4,5}[]

Order:  {2}[5], {3}[6], {5}[8], {4}[12]
Choose: {2}[5] ::  T = {2} since {2} is in I
Matroids

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Order: \{2\}[5], \{3\}[6], \{5\}[8], \{4\}[12]
Choose: \{2\}[5] :: T = \{2\} since \{2\} is in I
Choose: \{3\}[6] :: T = \{2,3\} since \{2,3\} is in I
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Matroids

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Matroids

But how does one do the test $T \cup \{e\} \in I$????
Matroids

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Interesting question since the size of the matroid is humongous!!!
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Depends on the particular problem!!
With computer structures there is always some way to tell very easily.
Matroids

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Interesting question since the size of the matroid is humongous!!!

Depends on the particular problem!!!
With computer structures there is always some way to tell very easily.

Let's consider some examples:

1. Minimum Cost Network:
   Given: a graph (vertices, edges) and costs on the edges
   Find: a least cost subset of edges s.t. for all pairs of vertices $<x,y>$ there is a path going solely through edges in the subset
Matroids

Is this a matroid???
Matroids

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What should the elements of E be?
Matroids

Is this a matroid???
  What should the elements of $E$ be?
    Right, edges!!!
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  What should the elements of $E$ be?
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  Are the appropriate subsets independent sets?
Matroids

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What should the elements of $E$ be?
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Are the appropriate subsets independent sets?
Let's see:

1. Is “For all $I_1 \in I$, if $I_2 \subset I_1$ then $I_2 \in I$.” good?
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Let's see:

1. Is “For all $I_1 \in I$, if $I_2 \subset I_1$ then $I_2 \in I.$” good?
   Each subset is a forest
Matroids

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2. Is “for all $I_1, I_2 \in I$ s.t. $|I_2| = |I_1| + 1$,
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Matroids

Three cases:
1. There is an edge of the larger subset which has at most one vertex that is an endpoint of an edge of the smaller set.
Matroids

Three cases:

2. There is an edge of the larger subset which has one endpoint in one connected component of the smaller subset and the other endpoint in another connected component of the smaller set.
Matroids

Three cases:

3. What's left: all edges of the larger subset are in the same connected component of the smaller subset
Matroids

So it is a matroid and the greedy method can be applied!!!

But how to check that $T \cup \{e\} \in I$?
Matroids

So it is a matroid and the greedy method can be applied!!!

But how to check that $T \cup \{e\} \in I$?

Need only check whether all the edges comprise a forest (that is, no cycles) – we discussed already ways to do this
Here is another example:

Integer Deadline Scheduling Problem:
  Given: Set of jobs, each with deadline and profit with unit processing time
  Find: A schedule of lowest cost such that total profit is maximized (no profit for job completed after its deadline)
Matroids

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Identify elements of E:
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This is a toughie!!!
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Identify elements of E:
  Right, the jobs!!

Identify the Independent sets:
  This is a toughie!!!
  Try this: Any set of jobs that can be scheduled so that all can be completed before their deadlines
Matroids

Is this a matroid???

1. Is “For all $I_1 \in I$, if $I_2 \subset I_1$ then $I_2 \in I.$” good?

<table>
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<tr>
<th>Job</th>
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<tbody>
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$I_1$

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$I_2$
Matroids

Is this a matroid???

2. Is “For all $I_1, I_2 \in I$ s.t. $|I_2| = |I_1| + 1$, 
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$I_2$  

$I_1$
Matroids

Is this a matroid???

2. Is “For all $I_1, I_2 \in I$ s.t. $|I_2| = |I_1| + 1$, 
   $\exists e \in I_2$ s.t. $I_1 \cup \{e\} \in I$” good?

<table>
<thead>
<tr>
<th>Job</th>
<th>Dead</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
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<td>501</td>
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<tr>
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<tr>
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<td>5</td>
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<tr>
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<td>4</td>
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<td>9</td>
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<td>112</td>
</tr>
</tbody>
</table>
Matroids

So what weight should we use for elements?
Matroids

So what weight should we use for elements?
Right, the profit
Matroids

So what weight should we use for elements?
Right, the profit

So how do we determine whether $T \cup \{e\} \in I$?
Matroids

So what weight should we use for elements?
Right, the profit

So how do we determine whether $T \cup \{e\} \in I$?
Right, keep 'em ordered by increasing deadline in $T$ and add 'em at the highest "open" deadline slot.