Why Do Engineers Need Math (and Matrices)?
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In the beginning there was light
and 100000000000000000000... interacting particles!
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How do you design a bridge, circuit, rocket if you have to worry about all those particles?
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You can’t! But a long time ago people figured out that the interactions are so predictable and uniform that they can be described abstractly using mathematics!
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So engineers use mathematics to model the real world abstractly. In other words - engineering is not hands on - it is MINDS ON.
**Why Do Engineers Need Math (and Matrices)?**

**Example:**

Define *voltage* as some mysterious force which we do not really understand very well, *current* as a rate of “flow” of some mysterious particles called *electrons*, and *resistance* as a property of imperfect materials which tends to restrict current.

A long time ago somebody figured out how to measure these phenomena and observed, via experiment, that

\[
\text{current} = \frac{\text{voltage}}{\text{resistance}}
\]

so it was called a law (Ohm’s law).
Why Do Engineers Need Math (and Matrices)?

Laws are great! 😊

From Ohm’s law we can build circuits!
From Newton’s laws we can build rockets!
There are laws for building bridges!
There are laws for remediating a polluted site!
There is conservation of energy!
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But Laws have side effects 😧

They aren’t laws - just first order approximations.
The context in which they are used must be understood.
Belief in laws inhibits creative advances.
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So, where does the math come in?

Laws are described as mathematical models.
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We can use math operations to manipulate the models and get answers to engineering questions.
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Laws are described as mathematical models.

We can use math operations to manipulate the models and get answers to engineering questions.

Useful operations: arithmetic, derivatives, integrals, matrix multiplication, eigenvalues and eigenvectors, probability, statistics.
Why Do Engineers Need Matrices?

Example 1: Visualize Contamination Plume (Env Eng)
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But how do we get these nice pictures from just a few well logs?
Example 1: Visualize Contamination Plume (Env Eng)
Overlay a grid, find values at each grid point.
Many ways: Inverse-distance-squared, Kriging, etc.
Why Do Engineers Need Matrices?

**Example 1:** Visualize Contamination Plume (Env Eng)

Once values are set for grid points, interpolate over edge of four edges of cell for contour value.

```
<table>
<thead>
<tr>
<th>5.78</th>
<th>6.47</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6.82</td>
<td>7.51</td>
</tr>
</tbody>
</table>
```
Example 1: Visualize Contamination Plume (Env Eng)
Once values are set for grid points, interpolate over edge of four edges of cell for contour value.
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**Example 1:** Visualize Contamination Plume (Env Eng)
Now connect all the straight lines from interpolation to get recognizable contours.
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Example 2: Predict Plume Movement
Given aquifer pressure, etc., and log data, predict the movement of the plume over time.

\[
R_f \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V_w \frac{\partial C}{\partial x}
\]

Boundary and Initial Conditions:

\[
C(x, t = 0) = 0 \quad \frac{\partial C}{\partial x}(x = \infty, t) = 0
\]

\[
C(x = 0, 0 < t \leq T) = C_0 \quad C(x = 0, t > T) = 0
\]

where \( R_f \) is a retardation coefficient, \( C \) is concentration of contaminant, \( D \) is the dispersion coefficient, and \( V_w \) is the velocity of the groundwater (one dimensional flow).
Example 2: Predict Plume Movement

2D: no symbolic solution - approximate over the grid

dby2 = b[i]+b[i-h_size];
ddX = dX/b[i];
ddY = dY/b[i];
X = 2*i;
Y = X+1;
s1 = (d[k]+d[k+3])*(un[i+1]-un[i])/dx;
s2 = (d[k+2]+d[k+5])*(un[i+h_size+1] - un[i-h_size+1]
   + un[i+h_size] - un[i-h_size])/(2.0*dy);
s3 = (d[k]+d[k-3])*(un[i]-un[i-1])/dx;
s4 = (d[k+2]+d[k-1])*(un[i+h_size] - un[i-h_size]
   + un[i+h_size-1] - un[i-h_size-1])/(2.0*dy);
s += ddX*(dbx1*(s1+s2) - dbx2*(s3+s4))/4.0;
s1 = (d[k+j+2]+d[k+2])*(un[i+h_size+1]-un[i+h_size-1]
   +un[i+1]-un[i-1])/(2.0*dx);
s2 = (d[k+j+1]+d[k+1])*(un[i+h_size] - un[i])/dy;
s3 = (d[k+2]+d[k-j+2])*(un[i+1]-un[i-1]+un[i-h_size+1]
   -un[i-h_size-1])/(2.0*dx);
s4 = (d[k+1]+d[k-j+1])*(un[i] - un[i-h_size])/dy;
s += ddY*(dby1*(s1+s2) - dby2*(s3+s4))/4.0;
une = (b[i]+b[i-1])*(un[i] - un[i-1])/2.;
unn = (b[i] + b[i-h_size])*(un[i] - un[i-h_size])/2.;
flow = f[i]*dt*un[i];
s += -flow/b[i] - v[X]*ddX*une - v[Y]*ddY*unn;
unp1[i] = s;
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Example 3: Remediation of Site
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**Example 4:** Classification (most Eng. disciplines)

<table>
<thead>
<tr>
<th>Data Point</th>
<th>Class</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>v</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>12.4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>10.2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>11.6</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>11.4</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>1000</td>
<td>0</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Determine the coefficients of

\[ Av + Bw + Cx + Dy + Ez \]

that maximize the number of correctly classified points
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Example 4: Classification (most Eng. disciplines)

Simple illustration in two parameters
Why Do Engineers Need Matrices?

Example 4: Classification (most Eng. disciplines)
Simple illustration in two parameters
Example 5: Decision Analysis/Optimization (all)

Schedule a conference

1. \( t \) periods
2. \( r \) parallel sessions
3. \( j^{th} \) room has capacity \( R_j \)
4. \( n \) speakers - \( i^{th} \) gives \( L_i \) lectures
5. \( m \) attendees - each with \( p \) preferences

Choose a schedule so that all attendees hear their preferred speakers
**Example 5:** Decision Analysis/Optimization (all)

Define $t \cdot r \cdot n$ variables

$$S_{i,j,k} = \begin{cases} 
1 & \text{if speaker } i \text{ is scheduled for room } j \text{ in period } k \\
0 & \text{otherwise}
\end{cases}$$

No two speakers speak in the same room at the same time.

$$S_{1,j,k} + S_{2,j,k} + S_{3,j,k} + \ldots + S_{n,j,k} \leq 1$$

$i^{th}$ speaker gives $L_i$ lectures.

$$S_{i,1,1} + \ldots + S_{i,1,t} + S_{i,2,1} + \ldots + S_{i,2,t} + \ldots + S_{i,r,1} + \ldots + S_{i,r,t} = L_i$$
**Example 5:** Decision Analysis/Optimization (all)

Problem is an example of matrix multiplication

\[
\begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
\end{pmatrix}
\cdot
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{pmatrix}
\leq
\begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
\end{pmatrix}
\]

where \(x_{k+(j-1)*t+(i-1)*t*r}\) corresponds to \(S_{i,j,k}\)
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Example 5: Decision Analysis/Optimization (all)

Programming the solution in MATLAB

```matlab
A = [1 1 0 0 ; 0 1 1 0 ; 0 0 1 1 ; 1 0 0 1 ; 0 1 1 0];
b = [1 ; 1 ; 1 ; 1 ; 1];
Aeq = [1 0 1 0 ; 0 1 0 1 ; 0 1 1 0];
beq = [1 ; 1 ; 2];
H = [-1 0 0 0 ; 0 -1 0 0 ; 0 0 -1 0 ; 0 0 0 -1];
f = [1 1 1 1];
lb = [0 ; 0 ; 0 ; 0];
ub = [1 ; 1 ; 1 ; 1];
x = quadprog(H,f,A,b,Aeq,beq,lb,ub);
```