Final Exam

Name: ___________________________  ID Number: ___________________________

Instructions: Answer all questions. Partial credit is considered if you state what you would do with an intermediate result if you were able to derive it.

1. (25) In the figure below draw the boundary of the region of points in $x_1, x_2$ space that satisfy the following constraints:

   \[
   \begin{align*}
   x_2 & \leq 1 \\
   x_1 & \leq 1 \\
   -x_1 - x_2 & \leq 0
   \end{align*}
   \]

   Now suppose we want to find values for $x_1$ and $x_2$ that minimize the objective function $x_1 - x_2$ and satisfy the above constraints. In the figure, draw the line representing the objective function so that it passes though the point corresponding to those values. What is the value of the objective function at that point?

   On the back of this page, write a Matlab function that finds the optimal values of $x_1$ and $x_2$. 
2. (2) What is the first thing to do when building a class?

(2) What is “local state” and where does it get defined?

(1) What is a method used for?

(20) A stack is a container of objects. Operations on a stack are insert and remove which add individual objects to and remove single objects from a stack, respectively. However, unlike in the case of a queue, the object removed is the one that was most recently inserted. Develop a stack class which supports insert and remove as above. Write all Matlab m files that are needed.
3. (25) You are given some number $n$ of processes. Each process has a deadline measured in seconds (assumed to be a positive integer no greater than $n$), and a profit (measured in U.S. dollars) and these numbers are also known. A processor will work on each process to completion, one at a time, according to a schedule. Each process takes exactly 1 second to complete. If a process is completed on or before its deadline, the person running the processor gets the profit of that process, otherwise the profit for that process is lost. Find the schedule that maximizes the profit for the person running the processor. For example consider the processes shown vertically in the following table:

<table>
<thead>
<tr>
<th>Process #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deadline</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Profit</td>
<td>23</td>
<td>19</td>
<td>32</td>
<td>17</td>
<td>21</td>
</tr>
</tbody>
</table>

Then processing in the order 5,1,2,3,4 gives a maximum total profit of 95 where only the profit of process 4 is lost. The problem can be solved several different ways and any good solution will be accepted. First, describe in English (below, on this page) how you would solve the problem for any given set of processes. Second, write something (on the next page) that seems to be a Matlab program using your ideas.
4. Variables $x_1, x_2, \ldots, x_n$ are random variables that each have probability 1/2 of taking value 1 and probability 1/2 of taking value 0.

(3) What is the mean of $x_1 + x_2 + \ldots + x_n$?

(3) What is the distribution of the sum $x_1 + x_2 + \ldots + x_n$?

(3) Does this look like another important distribution - how do you know?

(8) Design Matlab code to test this theory. First say in English what the code is supposed to do.

(8) Next write the code to do it