Instructions: Answer all questions. Partial credit is considered if you state what you would do with an intermediate result if you were able to derive it.

1. (32) Let $x_1, x_2, \ldots, x_n$ be $n$ variables. Write a system of linear and quadratic constraints, using $x_1, \ldots, x_n$ which, if satisfied, forces all variables to take a value of 0 or of 1, and no other value, and forces exactly three of the variables to have value 1 and all other variables to have value 0.
2. (33)

(a) Given \( n \) cities, in how many different ways can you pair two different cities?

(b) We want to lay cables between some pairs of cities so that every city connects to every other city through laid cables. Consider the following algorithm:

Put all pairings in a box.
 Arbitrarily and distinctly number the pairings.
 Repeat the following for each pairing, in increasing order of number:
 Remove the pairing from the box, call it \( P \).
 If a cable is laid for each pairing that remains in the box and some pair of cities is disconnected, put \( P \) back into the box.
 Lay a cable corresponding to each pairing in the box.

How many cables are laid upon completion of the algorithm in the case of \( n \) cities?

(c) How do you know all cities are connected by the laid cables?

(d) What MATLAB objects would you use to represent the box and the pairings?

(e) Describe how you would make the check called for in the If statement of the algorithm.
3. (35) A die is a cube with numbers from 1-6 appearing uniquely on each face. When a
die is rolled, the number of the top face is the value of the roll. The probability that
a particular number from 1-6 is the value of a single roll is 1/6 for a fair die.

(a) Suppose a die is rolled 6 times. What is the probability that 6 is the value of a
roll 1 time?

(b) Suppose a die is rolled 100 times. What is the mean sum of the values of all 100
rolls?

(c) Write a MATLAB function called \texttt{summit} that returns the sum of 100 numbers in
a given vector. Assume the function takes as input a vector of 100 integers from
1 to 6.

(d) Write a MATLAB function called \texttt{genit} that returns a vector of 100 numbers
each chosen uniformly and independently from the numbers 1 to 6.

(e) Write a MATLAB function called \texttt{doit} that uses \texttt{summit} and \texttt{genit} to check the
answer to question (b).
4. (35) We want to find out the best timing for a traffic light at a busy intersection at the peak of the rush hour. Two four lane roads cross at that intersection. Controlled directions through the intersection are North to South, South to North, East to West, West to East, and left turns North to East, West to North, South to West and East to South. The traffic light cycles according to the six diagrams of Figure 1 (in numerically increasing order). Assume that every second, with probability $P_{ns}$, a car arrives from the north intending to continue south, with probability $P_{ne}$ a car arrives from the north intending to turn east, and so on (that is, there are also probabilities $P_{sn}$, $P_{sw}$, $P_{ew}$, $P_{es}$, $P_{we}$, $P_{wn}$.) Assume that $S$ seconds must be allowed between changes for the intersection to clear (this time is not charged to directions that are not stopped at a light change - for example, this is not charged to direction North-South in changing from state 1 to state 2). Assume that the rate at which cars of a particular direction move across the intersection is given as $M$ per second, and $M$ is a positive number that is less than 1. Finally, define the frustration of a motorist by $c \cdot t^2$ where $t$ is the time spent waiting at the light and $c$ is some parameter. We want to establish the time, in seconds, that the traffic light is in each state of Figure 1 so as to minimize total frustration. We choose to use discrete event simulation for this task.

(a) What objects can you identify as needed to run such a simulation?

(b) What MATLAB objects would you use to represent these?

(c) Outline a solution in MATLAB. Identify functions as needed and say how they are implemented.
Figure 1: Traffic light cycle, left to right, top to bottom.