Balanced Binaries – AVL Trees
Balanced Binaries – AVL Trees

Height of a tree node:
1. The height of a node with no elements is 0
2. The height of a node with 1 element is 1
3. The height of a node with > 1 element is 1 + the height of its tallest subtree
Balanced Binaries – AVL Trees

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AVL tree:
A binary tree in which the difference between the height of the right and left subtrees of the root is never more than one.
Balanced Binaries – AVL Trees

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1. The height of a node with no elements is 0
2. The height of a node with 1 element is 1
3. The height of a node with > 1 element is 1 + the height of its tallest subtree

AVL tree:
A binary tree in which the difference between the height of the right and left subtrees of the root is never more than one.

Each node keeps a balance number which is the difference in heights of its two subtrees.

For example,

Whenever a balance number is not 0,-1,+1, perform some rotations according to some rules on following pages.
Balanced Binaries – AVL Trees

Rules for rotation:

If:

Plus mirror image of these two cases
Balanced Binaries – AVL Trees

Rules for rotation:

If:

\[
\begin{align*}
\text{If:} & \quad \begin{array}{c}
2 \\
\text{a} \\
-1 \\
\end{array} & \quad \begin{array}{c}
1 \\
\text{d} \\
\text{b} \\
\text{c} \\
\end{array} \\
\text{Then:} & \quad \begin{array}{c}
0 \\
\text{a} \\
\text{b} \\
\text{c} \\
\text{d} \\
\end{array}
\end{align*}
\]
Balanced Binaries – AVL Trees

Rules for rotation:

If:

Plus mirror image of these three cases
Balanced Binaries – AVL Trees

Example:
Balanced Binaries – AVL Trees

Example: Insert 2

changes
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Example:
Example: Insert 11
Balanced Binaries – AVL Trees

Example:
Balanced Binaries – AVL Trees

Example: Insert 9
Balanced Binaries – AVL Trees

Example: Insert 9

Rotation around 7
Balanced Binaries – AVL Trees

Example:
Balanced Binaries – AVL Trees

Example: Insert 6
Example: Insert 6

Double rotation
Balanced Binaries – AVL Trees

Example: Insert 6

Double rotation
Balanced Binaries – AVL Trees

Example:

```
  19
 /  \   
10   46
 / \   / \   
 4  14 37 55
 /   /   /   
 7 12 18 28 40 51
```

This diagram represents a balanced AVL tree, where each node is labeled with a value and the balance factor is indicated by the numbers associated with each node.
Balanced Binaries – AVL Trees

Example: Insert 56
Example: Insert 56

Single rotation around 58
Balanced Binaries – AVL Trees

Example:

```
    19
   /   \
 10    46
 /     /   \
 4    14   37
 /     /     /
 7    12   28
```

```
 0  0  0 
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Balanced Binaries – AVL Trees

Example: Insert 20
Example: Insert 20

Rotate around 28
Example: Insert 20

Rotate around 28
Example:
Balanced Binaries – AVL Trees

Example: Insert 23
Example: Insert 23

Rotation around 28
Balanced Binaries – AVL Trees

Example: Insert 23

Rotation around 28
Balanced Binaries – AVL Trees

Example:
Balanced Binaries – AVL Trees

Example: Insert 34
Balanced Binaries – AVL Trees

Example: Insert 34

Double rotation around 32
Example: Insert 34

Double rotation around 32
Example: Insert 34

Double rotation around 32
Example:
Example: Delete 18
Balanced Binaries – AVL Trees

Example: Delete 18

No change
Balanced Binaries – AVL Trees

Example:
Balanced Binaries – AVL Trees

Example: Delete 58

![AVL Tree Diagram](image-url)
Balanced Binaries – AVL Trees

Example: Delete 58

No change
Balanced Binaries – AVL Trees

Example:
Balanced Binaries – AVL Trees

Example: Delete 19
Balanced Binaries – AVL Trees

Example: Delete 19

```
              21
             /   \
            10    46
           /  \
          4   14   37
         /  \
        7   12  18   28
       /  \
      32  49  51   58
```

```
```
Example: Delete 21
Balanced Binaries – AVL Trees

Example: Delete 21

No change
Balanced Binaries – AVL Trees

Example: Delete 28

```
        28
       /   \
      10    46
     /     /  \
    4    14   37
   /  1   /  0
  7  12  18  32
```

```
Example: Delete 28

No change
Balanced Binaries – AVL Trees

Example: Delete 32
Example: Delete 32

Rotation around 55
Example: Delete 32

Rotation around 55