How many ways are there to choose three objects from a group of six?

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>145</td>
<td>146</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>124</td>
<td>245</td>
<td>246</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>134</td>
<td>345</td>
<td>346</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>234</td>
<td>126</td>
<td>156</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>136</td>
<td>256</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>236</td>
<td>356</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>235</td>
<td></td>
<td>456</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Can we calculate this?

1 2 3 4 5 6
Can we calculate this?

Choose squares one at a time
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position?
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6

How many numbers can we put in the second chosen position?
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6

How many numbers can we put in the second chosen position? 5
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6

How many numbers can we put in the second chosen position? 5

How many numbers can we put in the third chosen position?
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6
How many numbers can we put in the second chosen position? 5
How many numbers can we put in the third chosen position? 4
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6
How many numbers can we put in the second chosen position? 5
How many numbers can we put in the third chosen position? 4
Total number of ways to pull out 3 objects is $6 \times 5 \times 4 = 120$
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6
How many numbers can we put in the second chosen position? 5
How many numbers can we put in the third chosen position? 4
Total number of ways to pull out 3 objects is $6 \times 5 \times 4 = 120$

Huh? What’s wrong?
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6
How many numbers can we put in the second chosen position? 5
How many numbers can we put in the third chosen position? 4
Total number of ways to pull out 3 objects is \(6 \times 5 \times 4 = 120\)

Huh? What’s wrong?

1,2,3  1,3,2  2,1,3  2,3,1  3,1,2  3,2,1 should count once!

We need to divide 120 by \(3 \times 2 = 6\) to get 20 ways to choose 3 from 6
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6
How many numbers can we put in the second chosen position? 5
How many numbers can we put in the third chosen position? 4
Total number of ways to pull out 3 objects is $6 \times 5 \times 4 = 120$

Huh? What’s wrong?

1,2,3  1,3,2  2,1,3  2,3,1  3,1,2  3,2,1 should count once!

We need to divide 120 by $3 \times 2 = 6$ to get 20 ways to choose 3 from 6

This is $\frac{6!}{3!3!}$
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6

How many numbers can we put in the second chosen position? 5

How many numbers can we put in the third chosen position? 4

Total number of ways to pull out 3 objects is $6 \times 5 \times 4 = 120$

Huh? What’s wrong?

1,2,3 1,3,2 2,1,3 2,3,1 3,1,2 3,2,1 should count once!

We need to divide 120 by $3 \times 2 = 6$ to get 20 ways to choose 3 from 6

This is $\frac{6!}{3!3!}$ in general it’s $\frac{n!}{(n-m)!m!}$
Can we calculate this?

Choose squares one at a time

How many numbers can we put in the first chosen position? 6
How many numbers can we put in the second chosen position? 5
How many numbers can we put in the third chosen position? 4
Total number of ways to pull out 3 objects is $6 \times 5 \times 4 = 120$

Huh? What’s wrong?
1,2,3 1,3,2 2,1,3 2,3,1 3,1,2 3,2,1 should count once!

We need to divide 120 by $3 \times 2 = 6$ to get 20 ways to choose 3 from 6

This is $\frac{6!}{3!3!}$ in general it’s $\frac{n!}{(n-m)!m!}$ written as $\binom{n}{m}$
Can we calculate this?

What is the probability that one of the items selected is 1?
Can we calculate this?

What is the probability that one of the items selected is 1?

There is one way to choose the 1 - there are \(\binom{5}{2}\) ways to choose the rest.
Can we calculate this?

What is the probability that one of the items selected is 1?

There is one way to choose the 1 - there are \( \binom{5}{2} \) ways to choose the rest.

The probability is \( \frac{\binom{5}{2}}{\binom{3}{2}} = \frac{10}{20} = 0.5 \)
Can we calculate this?

What is the probability that one of the items selected is 1?

There is one way to choose the 1 - there are \( \binom{5}{2} \) ways to choose the rest.

The probability is \( \frac{\binom{5}{2}}{\binom{6}{3}} = \frac{10}{20} = 0.5 \)

What is the probability that two of the items selected are 1 and 2?
Can we calculate this?

What is the probability that one of the items selected is 1?

There is one way to choose the 1 - there are \( \binom{5}{2} \) ways to choose the rest.
The probability is \( \frac{\binom{5}{2}}{\binom{6}{3}} = \frac{10}{20} = 0.5 \)

What is the probability that two of the items selected are 1 and 2?

There is 1 way to choose 1 and 2 - there are \( \binom{4}{1} \) ways to choose the remaining number
Can we calculate this?

What is the probability that one of the items selected is 1?

There is one way to choose the 1 - there are \( \binom{5}{2} \) ways to choose the rest.

The probability is \( \frac{\binom{5}{2}}{\binom{6}{3}} = \frac{10}{20} = 0.5 \)

What is the probability that two of the items selected are 1 and 2?

There is 1 way to choose 1 and 2 - there are \( \binom{4}{1} \) ways to choose the remaining number

The probability is \( \frac{\binom{4}{1}}{\binom{6}{3}} = \frac{4}{20} = 0.2 \)
There are $n$ objects, $p$ of which are colored blue, the rest are colored red.

Pick a sample of $m$ objects.

Let $X$ be the number of objects in the sample that are colored blue.

$$Pr(X = k) = \binom{p}{k} \cdot \binom{n-p}{m-k} / \binom{n}{m}$$

$$\approx \binom{m}{k} \cdot \left(\frac{p}{n}\right)^k \cdot \left(1 - \frac{p}{n}\right)^{m-k}$$

This is a binominal distribution with $\mu = (p/n)m$ and variance $\sigma^2 = (p/n)m(1 - p/n)$. 
How to Compute $Pr(X = k) = \binom{p}{k} \cdot \binom{n-p}{m-k}/\binom{n}{m}$:

Start with $r=1$

Set $i = 0$

If $i \leq k-1$ do this:

Set $r = r \cdot ((p-i)/(n-m+k-i))$  \quad r = \frac{p}{n-m+k}$
How to Compute $Pr(X = k) = \binom{p}{k} \cdot \binom{n-p}{m-k} / \binom{n}{m}$:

Start with $r=1$

Set $i = 0$

If $i \leq k-1$ do this:

Set $r = r*\left(\frac{p-i}{n-m+k-i}\right)$

$r = \frac{p}{n-m+k}$

If $i \leq m-k-1$ do this:

Set $r = r*\left(\frac{n-p-i}{n-i}\right)*\left(\frac{m-i}{m-k-i}\right)$

$r = \frac{p(n-p)m}{(n-m+k)n(m-k)}$
How to Compute $Pr(X = k) = \left( \begin{array}{c} p \\ k \end{array} \right) \cdot \frac{(n-p)}{(m-k)} / \left( \begin{array}{c} n \\ m \end{array} \right)$:

Start with $r=1$

Set $i = 0$

If $i \leq k-1$ do this:

Set $r = r*\left( \frac{(p-i)}{(n-m+k-i)} \right)$  \quad r = \frac{p}{n-m+k}$

If $i \leq m-k-1$ do this:

Set $r = r*\left( \frac{(n-p-i)}{(n-i)} \right) * \left( \frac{(m-i)}{(m-k-i)} \right)$  \quad r = \frac{p(n-p)m}{(n-m+k)n(m-k)}$

Set $i = 1$

If $i \leq k-1$ do this:

Set $r = r*\left( \frac{(p-i)}{(n-m+k-i)} \right)$  \quad r = \frac{p(p-1)(n-p)m}{(n-m+k)(n-m+k-1)n(m-k)}$

If $i \leq m-k-1$ do this:

Set $r = r*\left( \frac{(n-p-i)}{(n-i)} \right) * \left( \frac{(m-i)}{(m-k-i)} \right)$  \quad r = \frac{p(p-1)(n-p)(n-p-1)m(m-1)}{(n-m+k)(n-m+k-1)n(n-1)(m-k)(m-k-1)}$
How to Compute $Pr(X = k) = \binom{p}{k} \cdot \frac{(n-p)}{m-k}/(n-m)$:

Start with $r=1$

Set $i = 0$

If $i \leq k-1$ do this:

Set $r = r * \left(\frac{p-i}{n-m+k-i}\right)$

$r = \frac{p}{n-m+k}$

If $i \leq m-k-1$ do this:

Set $r = r * \left(\frac{(n-p-i)}{(n-i)} \cdot \frac{(m-i)}{(m-k-i)}\right)$

$r = \frac{p(n-p)m}{(n-m+k)n(m-k)}$

Set $i = 1$

If $i \leq k-1$ do this:

Set $r = r * \left(\frac{p-i}{n-m+k-i}\right)$

$r = \frac{p(p-1)(n-p)m}{(n-m+k)(n-m+k-1)n(m-k)}$

If $i \leq m-k-1$ do this:

Set $r = r * \left(\frac{(n-p-i)}{(n-i)} \cdot \frac{(m-i)}{(m-k-i)}\right)$

$r = \frac{p(p-1)(n-p)(n-p-1)m(m-1)}{(n-m+k)(n-m+k-1)n(n-1)(m-k)(m-k-1)}$

...
How to Compute $Pr(X = k) = \binom{p}{k} \cdot \binom{n-p}{m-k}/\binom{n}{m}$:

Start with $r=1$

Set $i = 0$

If $i \leq k-1$ do this:

Set $r = r*\binom{p-i}{n-m+k-i} \quad r = \frac{p}{n-m+k}$

If $i \leq m-k-1$ do this:

Set $r = r*\binom{n-p-i}{n-i}*\binom{m-i}{m-k-i} \quad r = \frac{p(n-p)m}{(n-m+k)n(m-k)}$

Set $i = 1$

If $i \leq k-1$ do this:

Set $r = r*\binom{p-i}{n-m+k-i} \quad r = \frac{p(p-1)(n-p)m}{(n-m+k)(n-m+k-1)n(m-k)}$

If $i \leq m-k-1$ do this:

Set $r = r*\binom{n-p-i}{n-i}*\binom{m-i}{m-k-i} \quad r = \frac{p(p-1)(n-p)(n-p-1)m(m-1)}{(n-m+k)(n-m+k-1)n(n-1)(m-k)(m-k-1)}$

...
The following computes \( \binom{p}{k} \cdot \binom{n-p}{m-k}/\binom{n}{m} \).

%%% compute \((p\ choose\ k)\ast(n-p\ choose\ m-k)/(n\ choose\ m)\)

```matlab
function res = geomdist(n,p,m);
    res = zeros(1,m+1);
    for k=0:m
        r = 1;
        for i=0:max(k-1, m-k-1)
            if i <= k-1
                r = r*((p-i)/(n-m+k-i));
            end
            if i <= m-k-1
                r = r*((n-p-i)/(n-i))*((m-i)/(m-k-i));
            end
        end
        res(k+1) = r;
    end
end
```